Lagrangian mechanics of massless superparticle on $AdS_4 \times \mathbb{CP}^3$ superbackground

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Abstract

Massless superparticle model is considered on the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold and in the $AdS_4 \times \mathbb{CP}^3$ superspace. In the former case integrability of the equations of motion is rather obvious, while for the $AdS_4 \times \mathbb{CP}^3$ superparticle we prove integrability in the partial $\kappa-$symmetry gauge for which 4 anticommuting coordinates related to the broken conformal supersymmetry are set to zero. This allows us to propose expression for the Lax pair that may encode complete equations of motion for the $AdS_4 \times \mathbb{CP}^3$ superparticle.

1   Introduction

Aharony-Bergman-Jafferis-Maldacena (ABJM) correspondence [1] identifying $D = 3$ $\mathcal{N} = 6$ superconformal Chern-Simons-matter theory with $U(N)_k \times U(N)_{-k}$ gauge symmetry and M-theory on $AdS_4 \times (S^7/\mathbb{Z}_k)$ background belongs to the family of AdS/CFT dualities [2]. The distinctive feature of ABJM correspondence as compared to the AdS_5/CFT_4 one – the most notable member of the family – is non-maximal supersymmetry of the dual theories. So the $AdS_4 \times (S^7/\mathbb{Z}_k)$ background of $11d$ supergravity preserves 24 of 32 supersymmetries, the same amount of supersymmetry has the $AdS_4 \times \mathbb{CP}^3$ background of $D = 10$ IIA supergravity [3], [4], [5] that arises in the limit $k^5 \gg N \gg 1$. This fact limits straight-forward generalization of the results obtained for the AdS_5/CFT_4 correspondence to the ABJM duality case and issues new challenges.

Even the construction of the IIA superstring action on $AdS_4 \times \mathbb{CP}^3$ superbackground required non-trivial efforts. The group-theoretic supercoset approach [6], initially elaborated to construct the IIB superstring on $AdS_5 \times S^5$ superspace, gives only part of the $AdS_4 \times \mathbb{CP}^3$ superstring action that is described by the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model [7], [8][2]. Application of the supercoset approach is based on the isomorphism between the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold and the subspace of $AdS_4 \times \mathbb{CP}^3$ superspace parametrized by 10 space-time and 24 anticommuting coordinates associated with the unbroken supersymmetries of the background. The full-fledged $AdS_4 \times \mathbb{CP}^3$ superspace parametrized additionally by 8 Grassmann coordinates related to the broken space-time supersymmetries cannot be described as a supercoset manifold. A way to recover its supergeometry is the reduction of that for the maximally supersymmetric $AdS_4 \times S^7$ superspace, isomorphic to the $OSp(4|8)/(SO(1,3) \times SO(7))$ supercoset manifold, using the Hopf fibration realization of the 7-sphere $S^7 = \mathbb{CP}^3 \times S^1$ [4], [5]. Accordingly it was suggested in [7] that the complete $AdS_4 \times \mathbb{CP}^3$ superstring action can be obtained by the double-dimensional reduction [10], [11] of the $AdS_4 \times S^7$ supermembrane [12]. Geometric constituents of the $AdS_4 \times \mathbb{CP}^3$ superspace were found and applied to the construction of complete superstring action in

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[2]Alternative approach to constructing the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model action uses pure spinor variables [9].

[13][3]. Its structure, however, appears to be complicated highly non-linear. Attempts [14], [15] to simplify it by appropriate choice of gauges for local symmetries, in particular, the $\kappa-$symmetry have not resulted so far in a simple enough gauge-fixed action. The similar situation occurred with the $AdS_5 \times S^5$ superstring [16], [17], [18], [19].

The renewed impetus to exploring the AdS_5/CFT_4 correspondence was the discovery [20] of the integrability of $AdS_5 \times S^5$ superstring equations of motion supported by unveiling integrable structure in the dual $\mathcal{N} = 4$ super-Yang-Mills theory [21] (see [22] for a recent review). The integrability of equations of motion for the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model [7], [8] is the straight-forward generalization of that for the $AdS_5 \times S^5$ superstring[4]. This allowed to work out corresponding algebraic curve [26] and all-loop Bethe equations [27], [28].

Non-trivial problem is an extension of the integrable structure to the complete $AdS_4 \times \mathbb{CP}^3$ superstring case. Evidence supporting the possibility of such an extension was given in [29], [30]. Since the Lax connection encoding the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model equations takes value in the $osp(4|6)$ isometry superalgebra and is presented as the linear combination of $osp(4|6)$ Cartan forms it is reasonable to suggest that the Lax connection for the $AdS_4 \times \mathbb{CP}^3$ superstring is a generalization of that for the sigma-model by the contributions of fermionic coordinates related to 8 broken supersymmetries and their differentials. The authors of [30] succeeded in extending the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model Lax connection by linear and quadratic terms in those 'broken' fermions with its curvature turning to zero on the $AdS_4 \times \mathbb{CP}^3$ superstring equations up to quadratic order also. Obtained expression suggests that complete Lax connection for the $AdS_4 \times \mathbb{CP}^3$ superstring has a rather complicated form.

In [31] based on the isomorphism between the $osp(4|8)$ superalgebra and $D = 3$ $\mathcal{N} = 8$ superconformal algebra we proposed to use the $\kappa-$symmetry gauge freedom to single out 4 minimal $\frac{1}{4}$ fractions of broken supersymmetries described by $SL(2,\mathbb{R})$ (anti)Majorana fermions associated with the generators of broken Poincare and conformal supersymmetries.[5] This allows to introduce a minimal extension of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model. In the case when in the sector of broken supersymmetries there have been gauged away all the coordinates except for the $SL(2,\mathbb{R})$ Majorana spinor related to the broken Poincare supersymmetries[6] we have shown that the corresponding extension of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model is classically integrable with the Lax connection whose curvature strictly equals zero and that coincides with the subsector of the Lax connection found in [30]. Considering other $\kappa-$gauges that leave in the broken supersymmetries sector two of more $\frac{1}{4}$ fractions it might be possible to extend the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model Lax connection by corresponding $SL(2,\mathbb{R})$ (anti)Majorana coordinates. In particular, there are 6 ways to make up $\frac{1}{2}$ fraction of broken supersymmetries. One can choose anticommuting coordinates related to Poincare and conformal supersymmetries that satisfy

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[3]The $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model action comes about upon partial $\kappa-$symmetry gauge fixing of the $AdS_4 \times \mathbb{CP}^3$ superstring by setting to zero 8 Grassmann coordinates related to broken supersymmetries. This constrains the range of application of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model [7], [13].

[4]In fact both models belong to the class of integrable sigma-models on supercoset manifolds with the $\mathbb{Z}_4-$graded isometry superalgebras [23], [24], [25].

[5]Coordinates associated with the superconformal algebra generators were introduced in [32], [33], [34], [19] to describe the string/brane models related to the maximally supersymmetric examples of AdS/CFT correspondence.

[6]Allowing fermionic coordinates related to broken supersymmetries to be among the physical modes relaxes restrictions on the superstring motions that arise for the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model. For instance the string configurations lying within the AdS_4 part of the background that cannot be described by the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model can be considered in such a gauge.

(anti)Majorana condition. These two options are covered by the Kaluza-Klein gauge condition [31] characterized by vanishing of those summands in the $AdS_4 \times S^7$ supervielbein bosonic components in directions tangent to anti-de Sitter space-time that are proportional to the differential of the S^1 fiber coordinate dy. Two other possibilities correspond to choosing Majorana coordinate for the broken Poincare supersymmetries and anti-Majorana coordinate for the broken conformal supersymmetries and vice versa. Finally one can consider unconstrained $D = 3$ fermion θ^μ and its conjugate $\bar\theta^\mu$ associated with the generators of broken Poincare supersymmetries or unconstrained spinors η_μ and $\bar\eta_\mu$ related to the generators of broken conformal supersymmetries.[7] In these four latter cases the $AdS_4 \times S^7$ supervielbein bosonic components tangent to AdS_4 do have non-zero summands proportional to dy so that the tangent-space Lorentz rotation needs to be performed to remove them in accordance with the general prescription of the $D = 10$ supervielbein construction [10], [11].

So as a further step towards recovering the $AdS_4 \times \mathbb{CP}^3$ superstring integrable structure one can consider the extension of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model by the θ^μ, $\bar\theta^\mu$ Grassmann coordinates associated with the broken Poincare supersymmetries. This case combines reasonable simplicity with the possibility to probe those terms in the $AdS_4 \times \mathbb{CP}^3$ supervielbein that arise as a result of the Lorentz rotation. To simplify the matter as much as possible we concentrate on the zero modes sector described by the massless superparticle model.[8] As a warm up we start with the superparticle on the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset space introduced in Ref. [8], derive the equations of motion and find their Lax representation. Then we examine the case of superparticle on the $AdS_4 \times \mathbb{CP}^3$ superbackground with η_μ, $\bar\eta_\mu$ coordinates gauged away and obtain the Lax representation for the equations of motion in such a partial $\kappa-$symmetry gauge. This allows to make a prediction for the form of the Lax pair of the superparticle in complete $AdS_4 \times \mathbb{CP}^3$ superspace. We finish by discussing what can be learnt on the $AdS_4 \times \mathbb{CP}^3$ superstring Lax connection from the superparticle Lax pair.

2 Massless superparticle on the $OSp(4|6)/(SO(1,3) \times U(3))$ supermanifold

2.1 $osp(4|6)$ superalgebra, Cartan forms and $\mathbb{Z}_4-$grading

Taking an $OSp(4|6)/(SO(1,3) \times U(3))$ representative $\mathscr{G}$, left-invariant $osp(4|6)$ Cartan forms in conformal basis [35] can be grouped as

$$\mathscr{C}(d) = \mathscr{G}^{-1}d\mathscr{G} = \mathcal{C}_{\text{conf}} + \mathcal{C}_{\text{su}(4)} + \mathcal{C}_{24\text{susys}}. \tag{1}$$

The first summand introduces Cartan forms associated with the $D = 3$ conformal group generators (D, P_m, K_m, G_{mn})

$$\mathcal{C}_{\text{conf}} = \Delta(d)D + \omega^m(d)P_m + c^m(d)K_m + G^{mn}(d)M_{mn}. \tag{2}$$

Other bosonic Cartan forms are associated with the $so(6) \sim su(4)$ $R-$symmetry group generators

$$\mathcal{C}_{\text{su}(4)} = \Omega_a(d)T^a + \Omega^a(d)T_a + \widetilde\Omega_a{}^b(d)\widetilde{V}_b{}^a + \widetilde\Omega_a{}^a(d)\widetilde{V}_b{}^b. \tag{3}$$

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[7]These two options correspond to the $\kappa-$symmetry gauge conditions of Ref. [14] restricted to the 'broken' fermions.

[8]It can be viewed as the mass-to-zero limit of the D0-brane on $AdS_4 \times \mathbb{CP}^3$ superbackground [14].

They have been divided into $\widetilde{V}_b{}^a$ generators of the $U(3)$ stability group of $\mathbb{CP}^3 = SU(4)/U(3)$ manifold and the $su(4)/u(3)$ coset generators (T_a, T^a). The last summand in (1) includes odd Cartan forms

$$\mathcal{C}_{24\text{susys}} = \omega_a^\mu(d)Q_\mu^a + \bar{\omega}^{\mu a}(d)\bar{Q}_{\mu a} + \chi_{\mu a}(d)S^{\mu a} + \bar{\chi}_\mu^a(d)\bar{S}_a^\mu, \tag{4}$$

where the generators of $D = 3$ $\mathcal{N} = 6$ super-Poincare $(Q_\mu^a, \bar{Q}_{\mu a})$ and conformal $(S^{\mu a}, \bar{S}_a^\mu)$ supersymmetries carry $SL(2, \mathbb{R})$ spinor index $\mu = 1, 2$ and $SU(3)$ (anti)fundamental representation index $a = 1, 2, 3$ in accordance with the decomposition of $SO(6)$ vector on the $SU(3)$ representations $\mathbf{6} = \mathbf{3} \oplus \bar{\mathbf{3}}$.

(Anti)commutation relations of the $osp(4|6)$ superalgebra are known to be invariant under 4-element discrete automorphism Υ that assigns different eigenvalues to the $osp(4|6)$ generators

$$\Upsilon(g_{(k)}) = i^k g_{(k)}, \quad k = 0, 1, 2, 3: \quad [g_{(j)}, g_{(k)}\} = g_{(j+k)mod4}. \tag{5}$$

So that (1) can be written in the $\mathbb{Z}_4-$graded form

$$\mathscr{C}(d) = \mathscr{C}_{(0)}(d) + \mathscr{C}_{(2)}(d) + \mathscr{C}_{(1)}(d) + \mathscr{C}_{(3)}(d), \tag{6}$$

where

$$\begin{aligned}
\mathscr{C}_{(0)}(d) &= 2G^{3m}(d)M_{3m} + G^{mn}(d)M_{mn} + \widetilde{\Omega}_a{}^b(d)\widetilde{V}_b{}^a + \widetilde{\Omega}_a{}^a(d)\widetilde{V}_b{}^b \in g_{(0)}, \\
\mathscr{C}_{(2)}(d) &= 2G^{0'm}(d)M_{0'm} + \Delta(d)D + \Omega_a(d)T^a + \Omega^a(d)T_a \in g_{(2)}, \\
\mathscr{C}_{(1)}(d) &= \omega_{(1)a}^\mu(d)Q_{(1)\mu}^a + \bar{\omega}_{(1)}{}^{\mu a}(d)\bar{Q}_{(1)\mu a} \in g_{(1)}, \\
\mathscr{C}_{(3)}(d) &= \omega_{(3)a}^\mu(d)Q_{(3)\mu}^a + \bar{\omega}_{(3)}{}^{\mu a}(d)\bar{Q}_{(3)\mu a} \in g_{(3)}.
\end{aligned} \tag{7}$$

In the above formulae we introduced the $so(2,3)$ algebra generators $M_{0'm} = \frac{1}{2}(P_m + K_m)$, $M_{0'3} = -D$, $M_{3m} = \frac{1}{2}(K_m - P_m)$ and fermionic generators with definite $\mathbb{Z}_4-$grading

$$Q_{(1)\mu}^a = Q_\mu^a + iS_\mu^a, \quad \bar{Q}_{(1)\mu a} = \bar{Q}_{\mu a} - i\bar{S}_{\mu a}; \quad Q_{(3)\mu}^a = Q_\mu^a - iS_\mu^a, \quad \bar{Q}_{(3)\mu a} = \bar{Q}_{\mu a} + i\bar{S}_{\mu a}. \tag{8}$$

Associated bosonic

$$G^{0'm}(d) = \frac{1}{2}(\omega^m(d) + c^m(d)), \quad G^{3m}(d) = -\frac{1}{2}(\omega^m(d) - c^m(d)) \tag{9}$$

and fermionic Cartan forms

$$\omega_{(1)a}^\mu(d) = \frac{1}{2}(\omega_a^\mu(d) + i\chi_a^\mu(d)), \quad \omega_{(3)a}^\mu(d) = \frac{1}{2}(\omega_a^\mu(d) - i\chi_a^\mu(d)) \tag{10}$$

and c.c. are used to work out the Lax representation for superparticle equations of motion. Let us note that Cartan forms from $\mathscr{C}_{(2)}$ and $\mathscr{C}_{(1,3)}$ eigenspaces are identified with the bosonic and fermionic components of $OSp(4|6)/(SO(1,3) \times U(3))$ supervielbein, while those from $\mathscr{C}_{(0)}$ eigenspace describe $SO(1,3) \times U(3)$ connection.

2.2 $OSp(4|6)/(SO(1,3) \times U(3))$ superparticle action and equations of motion

Massless superparticle action on the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold [8]

$$\mathscr{S}_{\text{coset}} = \int \frac{d\tau}{e}(G_\tau{}^{0'}{}_m G_\tau{}^{0'm} + \Delta_\tau \Delta_\tau + \Omega_{\tau a}\Omega_\tau{}^a), \tag{11}$$

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constructed out of the world-line pullbacks of Cartan forms from the $\mathscr{C}_{(2)}$ eigenspace, is invariant under the $\mathbb{Z}_4$ automorphism and $OSp(4|6)$ global symmetry acting as the left group multiplication on $\mathscr{G}$. Action variation w.r.t. the Lagrange multiplier $e(\tau)$ produces the mass-shell constraint

$$G_\tau^{\,0'}{}_m G_\tau^{\,0'm} + \Delta_\tau \Delta_\tau + \Omega_{\tau a}\Omega_\tau{}^a = 0. \tag{12}$$

Since it is irrelevant to the subsequent discussion of the Lax representation for other (dynamical) equations of motion we set the Lagrange multiplier to unity. In this respect it can be said that we consider a '1d sigma model'.

To derive dynamical equations from (11) it is convenient to use the following general relation for the variation of a 1-form

$$\delta G(d) = d i_\delta G + i_\delta dG \tag{13}$$

and substitute in the second summand Maurer-Cartan equations for $\mathcal{C}_{(2)}$ Cartan forms [35]

$$
\begin{aligned}
dG^{0'm} - &\; 2G^{3m}(d) \wedge \Delta(d) - 2G^{mn}(d) \wedge G^{0'}{}_n(d) + 2i\omega_{(1)a}^{\mu}(d) \wedge \sigma_{\mu\nu}^m \bar{\omega}_{(1)}{}^{\nu a}(d) \\
+ &\; 2i\omega_{(3)a}^{\mu}(d) \wedge \sigma_{\mu\nu}^m \bar{\omega}_{(3)}{}^{\nu a}(d) = 0, \\
d\Delta + &\; 2G^{3m}(d) \wedge G^{0'}{}_m(d) + 2\omega_{(1)a}^{\mu}(d) \wedge \bar{\omega}_{(1)\mu}^{a}(d) - 2\omega_{(3)a}^{\mu}(d) \wedge \bar{\omega}_{(3)\mu}^{a}(d) = 0, \\
d\Omega^a + &\; i\Omega^b(d) \wedge (\widetilde{\Omega}_b{}^a(d) + \delta_b^a \widetilde{\Omega}_c{}^c(d)) - 2i\varepsilon^{abc}\omega_{(1)b}^{\mu}(d) \wedge \omega_{(1)\mu c}(d) \\
+ &\; 2i\varepsilon^{abc}\omega_{(3)b}^{\mu}(d) \wedge \omega_{(3)\mu c}(d) = 0.
\end{aligned}
\tag{14}
$$

Then taking the contractions of the Cartan forms (6) with the variation symbol i_δ as independent parameters yields the set of equations of motion

$$
\begin{aligned}
-\tfrac{1}{2}\frac{\delta\mathscr{S}_{\text{coset}}}{\delta G^{0'}{}_m(\delta)} &= \frac{dG_\tau^{\,0'm}}{d\tau} + 2G_\tau{}^{mn}G_\tau^{\,0'}{}_n + 2G_\tau{}^{3m}\Delta_\tau = 0, \\
-\tfrac{1}{2}\frac{\delta\mathscr{S}_{\text{coset}}}{\delta\Delta(\delta)} &= \frac{d\Delta_\tau}{d\tau} - 2G_\tau{}^{3m}G_\tau^{\,0'}{}_m = 0, \\
-\frac{\delta\mathscr{S}_{\text{coset}}}{\delta\Omega_a(\delta)} &= \frac{d\Omega_\tau{}^a}{d\tau} + i\Omega_\tau{}^b(\widetilde{\Omega}_{\tau b}{}^a + \delta_b^a \widetilde{\Omega}_{\tau c}{}^c) = 0; \\
-\tfrac{1}{4}\frac{\delta\mathscr{S}_{\text{coset}}}{\delta\bar{\omega}_{(1)}{}_\mu^a(\delta)} &= iG_\tau^{\,0'm}\tilde{\sigma}_m^{\mu\nu}\omega_{(1)\tau\nu a} + \Delta_\tau\omega_{(1)\tau a}^{\mu} - i\varepsilon_{abc}\Omega_\tau{}^b\bar{\omega}_{(1)\tau}^{\mu c} = 0; \\
-\tfrac{1}{4}\frac{\delta\mathscr{S}_{\text{coset}}}{\delta\bar{\omega}_{(3)}{}_\mu^a(\delta)} &= iG_\tau^{\,0'm}\tilde{\sigma}_m^{\mu\nu}\omega_{(3)\tau\nu a} - \Delta_\tau\omega_{(3)\tau a}^{\mu} + i\varepsilon_{abc}\Omega_\tau{}^b\bar{\omega}_{(3)\tau}^{\mu c} = 0
\end{aligned}
\tag{15}
$$

and c.c. equations[9].

2.3 Lax representation for the $OSp(4|6)/(SO(1,3) \times U(3))$ superparticle equations of motion

Eqs. (15) admit the Lax representation

$$\frac{d\mathfrak{l}}{d\tau} + [M, \mathfrak{l}] = 0 \tag{16}$$

with $M = \mathscr{C}_\tau$ determined by the pullback of Cartan forms (6) and

$$\mathfrak{l} = 2G_\tau^{\,0'm}M_{0'm} + \Delta_\tau D + \Omega_{\tau a}T^a + \Omega_\tau{}^a T_a \in g_{(2)}. \tag{17}$$

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[9]We assume right derivative for fermions.

Let us note the ambiguity in the definition of M. Instead of $\mathscr{C}_\tau$ it can be defined as $\mathscr{C}_{(0)\tau} \oplus \mathscr{C}_{(1)\tau} \oplus \mathscr{C}_{(3)\tau}$. This ambiguity is resolved when the 'broken' Grassmann coordinates are taken into consideration in favor of the first possibility, i.e. $M = \mathscr{C}_\tau$.

Since $\mathfrak{l}$ is determined by the Cartan forms that enter the superparticle action (11) one can write it down in the form of a differential operator from $g_{(2)}$ eigenspace acting on the action functional

$$\mathfrak{l} = \left(M_{0'm}\frac{\partial}{\partial G_\tau{}^{0'}{}_m} + \frac{1}{2}D\frac{\partial}{\partial \Delta_\tau} + T^a\frac{\partial}{\partial \Omega_\tau{}^a} + T_a\frac{\partial}{\partial \Omega_{\tau a}} \right) \mathscr{S}_{\text{coset}}. \tag{18}$$

Below we shall observe how this Lax pair generalizes by inclusion of the 'broken' fermions.

When the $OSp(4|6)/(SO(1,3) \times U(3))$ superparticle action is considered as arising from that of the superparticle on $AdS_4 \times \mathbb{CP}^3$ superbackground in the partial κ−symmetry gauge characterized by setting to zero 8 'broken' Grassmann coordinates there remain non-trivial equations for those coordinates

$$\Omega_{\tau a}\bar{\chi}_\tau{}^{\mu a} = 0, \quad \Omega_{\tau a}\bar{\omega}_\tau{}^{\mu a} = 0 \tag{19}$$

and c.c. They can be shown [36] to follow from the fermionic equations of the $OSp(4|6)/(SO(1,3) \times U(3))$ superparticle so that all the independent equation of motion are given by the system (15).[10] This is not the case for other choices of the κ−gauge when the fermionic coordinates associated with the broken supersymmetries are among the physical degrees of freedom.

3 Massless superparticle on the $AdS_4 \times \mathbb{CP}^3$ superbackground

3.1 $AdS_4 \times S^7$ superbackground, partial κ−symmetry gauge fixing and reduction to $D = 10$

Geometric constituents of the $AdS_4 \times \mathbb{CP}^3$ superspace parametrized by all 32 Grassmann coordinates can be derived from those of the $AdS_4 \times S^7$ superspace by means of the dimensional reduction [13], [14]. The maximally supersymmetric $AdS_4 \times S^7$ superspace is isomorphic to the $OSp(4|8)/(SO(1,3) \times SO(7))$ supercoset manifold with the $OSp(4|8)$ isometry supergroup. Hence the $D = 11$ supervielbein, connection and 4-form field strength are constructed out of the $osp(4|8)$ Cartan forms that can be presented in the $AdS_4 \times S^7$ or supermembrane basis as

$$\hat{\mathscr{G}}^{-1}d\hat{\mathscr{G}} = 2\mathcal{G}^{0'm'}(d)M_{0'm'} + \mathcal{G}^{m'n'}(d)M_{m'n'} + \Omega_{8I'}(d)V_{8I'} + \Omega_{I'J'}(d)V_{I'J'} + F^{\alpha A'}(d)O_{\alpha A'}, \tag{20}$$

where $\hat{\mathscr{G}}$ is an $OSp(4|8)/(SO(1,3) \times SO(7))$ representative. $so(2,3)/so(1,3) \oplus so(8)/so(7)$ Cartan forms are identified with the $AdS_4 \times S^7$ supervielbein bosonic components

$$\hat{E}^{m'}(d) = \mathcal{G}^{0'm'}(d), \quad \hat{E}_{I'}(d) = \Omega_{8I'}(d), \quad m' = 0,...,3; \ I' = 1,...,7 \tag{21}$$

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[10]Demonstrating that equations for the fermions associated with the broken supersymmetries constitute a part of those for the $OSp(4|6)/(SO(1,3) \times U(3))$ superparticle/sigma-model provides necessary consistency check of the corresponding κ−symmetry gauge condition. To leading order in the Grassmann coordinates of the $OSp(4|6)/(SO(1,3) \times U(3))$ supermanifold this was shown in [29] and the general proof was given in [36].

and fermionic Cartan forms – with the supervielbein fermionic components $F^{\alpha A'}(d)$ carrying indices $\alpha = 1, ..., 4$ of the Majorana spinor of $Spin(1,3)$ and $A' = 1, ..., 8$ of one of the $Spin(8)$ chiral spinor representations. Other Cartan forms describe the $so(1,3) \times so(7)$ connection. Field strength of the 3-form potential takes the form

$$H_{(4)} = \frac{i}{8} F^{\hat{\alpha}} \wedge \mathfrak{g}^{\hat{m}\hat{n}}{}_{\hat{\alpha}}{}^{\hat{\beta}} F_{\hat{\beta}} \wedge \hat{E}_{\hat{m}} \wedge \hat{E}_{\hat{n}} + \frac{1}{4} \varepsilon_{m'n'k'l'} \hat{E}^{m'} \wedge \hat{E}^{n'} \wedge \hat{E}^{k'} \wedge \hat{E}^{l'}, \tag{22}$$

where $\mathfrak{g}^{\hat{m}\hat{n}}{}_{\hat{\alpha}}{}^{\hat{\beta}}$ stands for the antisymmetrized product of $D = 11$ gamma-matrices $\mathfrak{g}^{\hat{m}}{}_{\hat{\alpha}}{}^{\hat{\beta}}$ with $\hat{m} = m' \oplus I'$ and $\hat{\alpha} = \alpha A'$, $\hat{\beta} = \beta B'$ being vector and spinor indices.[11] $D = 11$ supervielbein bosonic components (21) and 4-form field strength (22) enter the $AdS_4 \times S^7$ supermembrane action [12] that can be dimensionally reduced to the Type IIA superstring action on $AdS_4 \times \mathbb{CP}^3$ superspace [13].

Dimensional reduction to the $AdS_4 \times \mathbb{CP}^3$ superspace is based on the Hopf fibration realization of the 7-sphere $S^7 = \mathbb{CP}^3 \times S^1$ [4], [5]. It requires identifying mutually commuting $su(4)$ and $u(1)$ isometry algebras of $\mathbb{CP}^3$ and S^1 respectively within the $so(8)$ algebra of S^7. To this end the change of basis for the $so(8)$ generators $(V_{8I'}, V_{I'J'})$ needs to be performed [13], [15]. The $su(4) \sim so(6)$ becomes the bosonic subalgebra of $osp(4|6)$ isometry superalgebra of the $AdS_4 \times \mathbb{CP}^3$ superbackground. 24 Fermionic generators of $osp(4|6)$ superalgebra are in one-to-one correspondence with the space-time supersymmetries of $AdS_4 \times \mathbb{CP}^3$ superspace. They, as well as the generators of 8 supersymmetries broken by the $AdS_4 \times \mathbb{CP}^3$ superbackground, can be extracted from 32 fermionic generators of $osp(4|8)$ superalgebra by acting with the projection matrices [4], [13], whose form is determined by the Kähler 2-form of $\mathbb{CP}^3$. Particularly convenient realization for that tensor, in which these projectors diagonalize, is in terms of $D = 6$ chiral gamma-matrices [37]. Further using the isomorphism between the $osp(4|8)$ superalgebra and $D = 3$ $\mathcal{N} = 8$ superconformal algebra [15], decomposing $D = 6$ vectors and $D = 8$ spinors under $SU(3)$ as $\mathbf{6} = \mathbf{3} \oplus \bar{\mathbf{3}}$ and $\mathbf{8} = \mathbf{4} \oplus \bar{\mathbf{4}} = (\mathbf{3} \oplus \mathbf{1}) \oplus (\bar{\mathbf{3}} \oplus \bar{\mathbf{1}})$ allows to present (20) as

$$\hat{\mathscr{G}}^{-1} d\hat{\mathscr{G}} = \mathcal{C}_{\underline{\mathrm{conf}}} + \mathcal{C}_{so(8)} + \mathcal{C}_{32\mathrm{susys}}. \tag{23}$$

The first summand

$$\mathcal{C}_{\underline{\mathrm{conf}}} = \underline{\Delta}(d)D + \underline{\omega}^m(d)P_m + \underline{c}^m(d)K_m + \underline{G}^{mn}(d)M_{mn} \tag{24}$$

contains Cartan forms related to $D = 3$ conformal group generators similarly to (2). In the above and subsequent relations we underline those $osp(4|6)$ Cartan forms that acquire dependence on 8 Grassmann coordinates associated with the broken supersymmetries and the S^1 coordinate differential dy in addition to the coordinates parametrizing the $OSp(4|6)/(SO(1,3) \times U(3))$ manifold. The second summand

$$\begin{aligned} \mathcal{C}_{so(8)} = \ & \Omega_a(d)T^a + \Omega^a(d)T_a + \widetilde{\Omega}_a(d)\widetilde{T}^a + \widetilde{\Omega}^a(d)\widetilde{T}_a \\ & + \ \widetilde{\Omega}_a{}^b(d)\widetilde{V}_b{}^a + \widetilde{\Omega}_b{}^a(d)\widetilde{V}_a{}^a + \Omega_a{}^4(d)V_4{}^a + \Omega_4{}^a(d)V_a{}^4 + h(d)H \end{aligned} \tag{25}$$

introduces Cartan forms for the $so(8)$ generators in the basis adapted for the Hopf fibration realization of the 7-sphere. The generators $(T_a, T^a, \widetilde{V}_b{}^a)$ form the $su(4) \subset so(8)$ algebra (cf. (3)). The generator H corresponds to the $u(1)$ isometry of S^1 commuting with $su(4)$, while remaining 12 generators parametrize the $so(8)/(su(4) \times u(1))$ coset. Explicit expression for those generators in terms of $V_{8I'} = (V_{8a}, V_8{}^a, V_{87})$ and $V_{I'J'} = (V_{7a}, V_7{}^a; V_a{}^b, V_a{}^4, V_4{}^a)$ is

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[11]More details on the notation and spinor algebra can be found in [15].

given by the relations [15], [31] [12]

$$T_a = \tfrac{1}{2}(V_{7a} - iV_{8a}), \quad T^a = -\tfrac{1}{2}(V_7{}^a + iV_8{}^a), \quad \widetilde{V}_a{}^b = V_a{}^b - \tfrac{1}{2}\delta_a^b V_c{}^c + \tfrac{1}{4}\delta_a^b V_{87};$$
$$H = V_{87} + 2V_a{}^a; \quad \widetilde{T}_a = -\tfrac{1}{2}(V_{7a} + iV_{8a}), \quad \widetilde{T}^a = \tfrac{1}{2}(V_7{}^a - iV_8{}^a). \tag{26}$$

Accordingly Cartan forms in this basis are expressed through $\Omega_{8I'} = (\Omega_{8a},\ \Omega_8{}^a,\ \Omega_{87})$ and $\Omega_{I'J'} = (\Omega_{7a},\ \Omega_7{}^a; \Omega_a{}^b,\ \Omega_a{}^4,\ \Omega_4{}^a)$ that enter (20)

$$\Omega_a(d) = \Omega_{7a} - \tfrac{i}{2}\Omega_{8a}, \quad \widetilde{\Omega}_a(d) = -\Omega_{7a} - \tfrac{i}{2}\Omega_{8a},$$
$$\Omega^a(d) = -\Omega^{7a} - \tfrac{i}{2}\Omega^{8a}, \quad \widetilde{\Omega}^a(d) = \Omega^{7a} - \tfrac{i}{2}\Omega^{8a}, \tag{27}$$
$$h(d) = \tfrac{1}{4}(\Omega_{87} + 2\Omega_a{}^a), \quad \widetilde{\Omega}_a{}^b(d) = \Omega_a{}^b - \tfrac{1}{2}\delta_a^b \Omega_c{}^c + \tfrac{1}{4}\delta_a^b \Omega_{87}.$$

Finally $\mathcal{C}_{32\mathrm{susys}}$ is the linear combination of odd generators divided into those corresponding to unbroken $(Q_\mu^a,\ \bar{Q}_{\mu a};\ S^{\mu a},\ \bar{S}_a^\mu)$ and broken $(Q_\mu^4,\ \bar{Q}_{\mu 4};\ S^{\mu 4},\ \bar{S}_4^\mu)$ supersymmetries from the $AdS_4 \times \mathbb{CP}^3$ superbackground perspective

$$\mathcal{C}_{32\mathrm{susys}} = \quad \underline{\omega}_a^\mu(d)Q_\mu^a + \bar{\underline{\omega}}^{\mu a}(d)\bar{Q}_{\mu a} + \omega_4^\mu(d)Q_\mu^4 + \bar{\omega}^{\mu 4}(d)\bar{Q}_{\mu 4}$$
$$+ \quad \underline{\chi}_{\mu a}(d)S^{\mu a} + \bar{\underline{\chi}}_\mu^a(d)\bar{S}_a^\mu + \chi_{\mu 4}(d)S^{\mu 4} + \bar{\chi}_\mu^4(d)\bar{S}_4^\mu. \tag{28}$$

Then the $D = 11$ supervielbein bosonic components (21) are expressed through the Cartan forms (24) and (25)

$$\hat{E}^{m'}(d) = \left(\tfrac{1}{2}(\underline{\omega}^m(d) + \underline{c}^m(d)),\ -\underline{\Delta}(d)\right),$$
$$\hat{E}_{I'}(d) = \left(i(\Omega_a(d) + \widetilde{\Omega}_a(d)),\ i(\Omega^a(d) + \widetilde{\Omega}^a(d)),\ h(d) + \widetilde{\Omega}_a{}^a(d)\right). \tag{29}$$

Similarly supervielbein fermionic components coincide with the Cartan forms (28).

Anticipating reduction to $D = 10$ explicit expressions for the $AdS_4 \times S^7$ supervielbein components can be conveniently written in terms of the $osp(4|6)$ Cartan forms, the differential of S^1 Hopf fiber coordinate dy and 8 Grassmann coordinates $(\theta^\mu,\ \bar{\theta}^\mu,\ \eta_\mu,\ \bar{\eta}_\mu)$ for the supersymmetries broken by the $AdS_4 \times \mathbb{CP}^3$ superbackground. The $OSp(4|8)/(SO(1,3) \times SO(7))$ representative $\hat{\mathscr{G}}$ can be taken in the form of a $OSp(4|6)/(SO(1,3) \times U(3))$ element 'dressed' by the S^1 Hopf fiber coordinate and 8 'broken' fermions, e.g. as in [15], [31]

$$\hat{\mathscr{G}} = \mathscr{G}\, e^{yH} e^{\theta^\mu Q_\mu^4 + \bar{\theta}^\mu \bar{Q}_{\mu 4}} e^{\eta_\mu S^{\mu 4} + \bar{\eta}_\mu \bar{S}_4^\mu} \in OSp(4|8)/(SO(1,3) \times SO(7)) \tag{30}$$

that ensures the supervielbein independence on y, however, contributions proportional to dy do arise for fermionic components of the supervielbein and bosonic ones in directions tangent to the AdS_4 space-time because of the structure of underlying $osp(4|8)$ superalgebra.

Considerable simplification of the $AdS_4 \times S^7$ supervielbein can be attained by partially fixing $\kappa-$symmetry gauge freedom in the broken supersymmetries sector. In [31] we have proposed the Kaluza-Klein $\kappa-$gauge characterized by vanishing of contributions proportional to dy in the $D = 11$ supervielbein bosonic components that are tangent to $D = 10$ space-time. Another possibility of simplifying the $AdS_4 \times S^7$ supergeometry is to set to zero

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[12] In Ref. [13] the $so(8)$ generators adapted for the S^7 Hopf fibration were given in conventional $D = 6$ vector basis and without specifying explicitly the Kähler 2-form on $\mathbb{CP}^3$. However, natural realization of the $u(3)$ stability algebra of $\mathbb{CP}^3$ is provided by the generators $\widetilde{V}_a{}^b$ in $\mathbf{3} \times \bar{\mathbf{3}}$ representation of $SU(3)$. Associated representation of $D = 6$ vectors in $\mathbf{3} \oplus \bar{\mathbf{3}}$ basis suggests the choice of the Kähler 2-form such that its contraction with the $su(4)$ generators constructed out of $D = 6$ chiral gamma-matrices is given by the diagonal 4×4 matrix [37].

Grassmann coordinates η_μ, $\bar\eta_\mu$ associated with the broken part of $D = 3$ $\mathcal{N} = 8$ conformal supersymmetry. In such a partial κ−symmetry gauge the $AdS_4 \times S^7$ supervielbein bosonic components acquire the form[13]

$$\hat{E}^m(d) = \mathfrak{e}^m(d) + \tfrac{1}{4}c^m(d)\theta^2\bar\theta^2 + dyG_y^m, \quad \hat{E}^{11}(d) = dy + \widetilde{\Omega}_a{}^a(d) + c^m(d)(\theta\sigma_m\bar\theta);$$
$$E^3(d) = -\Delta(d), \quad E_a(d) = i(\Omega_a(d) + 2\chi_{\mu a}(d)\theta^\mu), \quad E^a(d) = i(\Omega^a(d) - 2\bar\chi_\mu^a(d)\bar\theta^\mu),$$

<div align="right">(31)</div>

where

$$\mathfrak{e}^m(d) = G^{0'm}(d) - \frac{i}{2}\left(d\theta\sigma^m\bar\theta + d\bar\theta\sigma^m\theta + \varepsilon^{mkl}G_{kl}(d)(\theta\bar\theta)\right) \tag{32}$$

and

$$G_y^m = 2(\theta\sigma^m\bar\theta). \tag{33}$$

We adopt the following notation for contractions of 2-component spinors $(\theta\sigma^m\bar\theta) = \theta^\mu\sigma^m_{\mu\nu}\bar\theta^\nu$, $(\theta\bar\theta) = \theta^\mu\bar\theta_\mu$ etc. The presence of contributions proportional to dy necessitates the $SO(1,3)$ Lorentz rotation to be performed in directions tangent to the Minkowski boundary of AdS_4 and S^1

$$(L\hat{E})^m(d) = L^m{}_n\hat{E}^n + L^m{}_{11}\hat{E}^{11}$$
$$(L\hat{E})^{11}(d) = L^{11}{}_m\hat{E}^m + L^{11}{}_{11}\hat{E}^{11} = \Phi_L(dy + A_L),$$

<div align="right">(34)</div>

where $\Phi_L = e^{2\phi/3}$ is related to the $D = 10$ dilaton ϕ and A_L is identified with the RR 1-form potential. Lorentz rotation matrix

$$||L|| = \begin{pmatrix} L^m{}_n & L^m{}_{11} \\ L^{11}{}_m & L^{11}{}_{11} \end{pmatrix} \in SO(1,3) \tag{35}$$

is defined by the requirement of removing the summand proportional to dy in $(L\hat{E})^m$, i.e. $L^m{}_nG_y^n + L^m{}_{11} = 0$. Explicit form of its entries is as follows

$$L^m{}_n = (1 - \theta^2\bar\theta^2)\delta_n^m, \quad L^m{}_{11} = -2(\theta\sigma^m\bar\theta),$$
$$L^{11}{}_m = 2(\theta\sigma_m\bar\theta), \quad L^{11}{}_{11} = 1 - 3\theta^2\bar\theta^2.$$

<div align="right">(36)</div>

Then $(L\hat{E})^m$ is identified with the $D = 10$ supervielbein bosonic components E^m in directions tangent to the Minkowski boundary of AdS_4 space-time

$$E^m(d) = \mathfrak{e}^m(d) - 2\widetilde{\Omega}_a{}^a(d)(\theta\sigma^m\bar\theta) - \frac{7}{4}G^{0'm}(d)\theta^2\bar\theta^2 - \frac{3}{4}G^{3m}(d)\theta^2\bar\theta^2. \tag{37}$$

The scalar superfield is given by the following expression

$$\Phi_L = 1 + 3\theta^2\bar\theta^2. \tag{38}$$

Other bosonic components of the $D = 11$ supervielbein $E^3(d)$, $E_a(d)$ and $E^a(d)$ are not affected by the Lorentz rotation and can be directly identified with the $D = 10$ supervielbein bosonic components in the corresponding κ−symmetry gauge[14].

---

[13]In what follows concentrating on the particular choice of the κ−symmetry gauge we retain the same notation for the gauge-fixed expressions for $D = 11$ and $D = 10$ supervielbein components.

[14]Fermionic supervielbein components are transformed under the spinor version of the above Lorentz rotation (see [13], [14], [15]). We do not discuss it here since the superparticle action is constructed out the $D = 10$ supervielbein bosonic components only.

3.2 $AdS_4 \times \mathbb{CP}^3$ superparticle action and equations of motion in the partial $\kappa-$symmetry gauge

Superparticle action on the $AdS_4 \times \mathbb{CP}^3$ superbackground

$$\mathscr{S} = \int \frac{d\tau}{e} \Phi_L \left(E_{\tau m} E_\tau^m + E_\tau^3 E_\tau^3 - E_{\tau a} E_\tau{}^a \right) \tag{39}$$

in considered partial $\kappa-$symmetry gauge by substituting expressions for the gauge-fixed supervielbein bosonic components (37) and (31) is brought to the form

$$\mathscr{S}_{\text{g.f.}} = \int \frac{d\tau}{e} \left[\mathfrak{e}_{\tau m} \mathfrak{e}_\tau^m - 4\widetilde{\Omega}_{\tau a}{}^a \mathfrak{e}_{\tau m} (\theta \sigma^m \bar\theta) + 6\widetilde{\Omega}_{\tau a}{}^a \widetilde{\Omega}_{\tau b}{}^b \theta^2 \bar\theta^2 - \tfrac{1}{2} G_\tau{}^{0'}{}_m G_\tau{}^{0'm} \theta^2 \bar\theta^2 \right. \tag{40}$$
$$\left. - \tfrac{3}{2} G_\tau{}^{0'}{}_m G_\tau{}^{3m} \theta^2 \bar\theta^2 + \Phi_L (\Delta_\tau \Delta_\tau - E_{\tau a} E_\tau{}^a) \right].$$

To derive equations of motion following from the action (40) analogously to the case of superparticle on the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold it is helpful to consider as variation parameters the $osp(4|6)$ Cartan forms with definite $\mathbb{Z}_4-$grading (6). Also we set the Lagrange multiplier e to unity since the mass-shell constraint is not involved in the Lax representation for the equations of motion. In such a way it is possible to get the set of bosonic equations of motion

$$-\tfrac{1}{2} \frac{\delta \mathscr{S}_{\text{g.f.}}}{\delta G^{0'}{}_m(\delta)} = \frac{d}{d\tau} \mathfrak{E}_\tau{}^m + 2G_\tau{}^m{}_n \mathfrak{E}_\tau{}^n + 2\Phi_L G_\tau{}^{3m} \Delta_\tau + iE_{\tau a} (\bar\omega_\tau^a \sigma^m \bar\theta) - iE_\tau{}^a (\omega_{\tau a} \sigma^m \theta)$$
$$+ 2i\varepsilon^{mkl} \mathfrak{e}_{\tau k} G_\tau{}^{0'}{}_l (\theta\bar\theta) - \tfrac{3}{2} \Delta_\tau G_\tau{}^{0'm} \theta^2 \bar\theta^2 = 0,$$

$$-\tfrac{1}{2} \frac{\delta \mathscr{S}_{\text{g.f.}}}{\delta \Delta(\delta)} = \frac{d}{d\tau} (\Phi_L \Delta_\tau) - 2G_{\tau 3m} \mathfrak{E}_\tau{}^m + iE_{\tau a} \bar\chi_\tau{}^{\mu a} \bar\theta_\mu - iE_\tau{}^a \chi_{\tau a}{}^\mu \theta_\mu$$
$$+ \tfrac{3}{2} G_\tau{}^{0'}{}_m G_\tau{}^{0'm} \theta^2 \bar\theta^2 = 0, \tag{41}$$

$$-i \frac{\delta \mathscr{S}_{\text{g.f.}}}{\delta \Omega_a(\delta)} = \frac{d}{d\tau} (\Phi_L E_\tau{}^a) + i\Phi_L E_\tau{}^b (\widetilde{\Omega}_{\tau b}{}^a + \delta_b^a \widetilde{\Omega}_{\tau c}{}^c) - 4\Omega_\tau{}^a \mathfrak{E}_\tau{}^m (\theta \sigma_m \bar\theta)$$
$$- 2i\varepsilon^{abc} E_{\tau b} \chi_{\tau c}{}^\mu \bar\theta_\mu = 0,$$

$$-i \frac{\delta \mathscr{S}_{\text{g.f.}}}{\delta \Omega^a(\delta)} = \frac{d}{d\tau} (\Phi_L E_{\tau a}) - i(\widetilde{\Omega}_{\tau a}{}^b + \delta_a^b \widetilde{\Omega}_{\tau c}{}^c) \Phi_L E_{\tau b} + 4\Omega_{\tau a} \mathfrak{E}_\tau{}^m (\theta \sigma_m \bar\theta)$$
$$- 2i\varepsilon_{abc} E_\tau{}^b \bar\chi_\tau{}^{\mu c} \theta_\mu = 0,$$

where

$$\mathfrak{E}_\tau{}^m = \Phi_L E_\tau{}^m - \frac{7}{4} G_\tau{}^{0'm} \theta^2 \bar\theta^2 \tag{42}$$

and $D = 10$ supervielbein components tangent to the $\mathbb{CP}^3$ manifold $E_a(d)$, $E^a(d)$ are given in (31). Analogously fermionic equations arise from the variation w.r.t. $\mathbb{Z}_4-$graded fermionic $osp(4|6)$ Cartan forms

$$-\tfrac{1}{4} \frac{\delta \mathscr{S}_{\text{g.f.}}}{\delta \bar\omega_{(1)}{}^a_\mu(\delta)} = i\mathfrak{E}_\tau{}^m \tilde\sigma_m^{\mu\nu} \omega_{(1)\tau\nu a} + \Phi_L \left[\Delta_\tau \omega_{(1)\tau a}{}^\mu - \varepsilon_{abc} E_\tau{}^b (\bar\omega_{(1)\tau}{}^{\mu c} + i\theta^\mu \bar\chi_{\tau\nu}{}^c \bar\theta^\nu) \right]$$
$$+ \tfrac{1}{2} \frac{d}{d\tau} (E_{\tau a} \bar\theta^\mu) - \tfrac{i}{2} \left(\widetilde{\Omega}_{\tau a}{}^b - \delta_a^b \widetilde{\Omega}_{\tau c}{}^c \right) E_{\tau b} \bar\theta^\mu + \tfrac{1}{4} E_{\tau a} \left[G_\tau{}^{mn} \bar\theta^\nu \sigma_{mn\nu}{}^\mu \right] \tag{43}$$
$$+ 2\Delta_\tau \bar\theta^\mu + 2i(G_\tau{}^{0'm} + G_\tau{}^{3m}) \tilde\sigma_m^{\mu\nu} \bar\theta_\nu \right] - i\mathfrak{e}_\tau{}^m \tilde\sigma_m^{\mu\nu} \omega_{(3)\tau\nu a} (\theta\bar\theta)$$
$$- 2i\omega_{(3)\tau a}{}^\mu \mathfrak{E}_\tau{}^m (\theta \sigma_m \bar\theta) + \tfrac{3i}{4} G_\tau{}^{0'm} \tilde\sigma_m^{\mu\nu} \omega_{(3)\tau\nu a} \theta^2 \bar\theta^2 = 0;$$

$$-\tfrac{1}{4} \frac{\delta \mathscr{S}_{\text{g.f.}}}{\delta \bar\omega_{(3)}{}^a_\mu(\delta)} = i\mathfrak{E}_\tau{}^m \tilde\sigma_m^{\mu\nu} \omega_{(3)\tau\nu a} - \Phi_L \left[\Delta_\tau \omega_{(3)\tau a}{}^\mu - \varepsilon_{abc} E_\tau{}^b (\bar\omega_{(3)\tau}{}^{\mu c} + i\theta^\mu \bar\chi_{\tau\nu}{}^c \bar\theta^\nu) \right]$$
$$- \tfrac{1}{2} \frac{d}{d\tau} (E_{\tau a} \bar\theta^\mu) + \tfrac{i}{2} \left(\widetilde{\Omega}_{\tau a}{}^b - \delta_a^b \widetilde{\Omega}_{\tau c}{}^c \right) E_{\tau b} \bar\theta^\mu - \tfrac{1}{4} E_{\tau a} \left[G_\tau{}^{mn} \bar\theta^\nu \sigma_{mn\nu}{}^\mu \right] \tag{44}$$
$$+ 2\Delta_\tau \bar\theta^\mu - 2i(G_\tau{}^{0'm} + G_\tau{}^{3m}) \tilde\sigma_m^{\mu\nu} \bar\theta_\nu \right] + i\mathfrak{e}_\tau{}^m \tilde\sigma_m^{\mu\nu} \omega_{(1)\tau\nu a} (\theta\bar\theta)$$
$$+ 2i\omega_{(1)\tau a}{}^\mu \mathfrak{E}_\tau{}^m (\theta \sigma_m \bar\theta) + \tfrac{3i}{4} G_\tau{}^{0'm} \tilde\sigma_m^{\mu\nu} \omega_{(1)\tau\nu a} \theta^2 \bar\theta^2 = 0.$$

Observe that Eqs. (41)-(44) reduce to the equations of motion (15) for the superparticle on the $OSp(4|6)/(SO(1,3) \times U(3))$ supermanifold when Grassmann coordinates θ^μ, $\bar{\theta}^\mu$ are set to zero. There are also 8 fermionic equations associated with the broken supersymmetries

$$
\begin{aligned}
\frac{\delta \mathscr{S}_{\text{g.f.}}}{\delta \bar{\theta}_\mu} = & -2i E_{\tau a} \bar{\chi}_\tau{}^{\mu a} - i \tfrac{d}{d\tau} (\mathfrak{E}_\tau{}^m \tilde{\sigma}_m^{\mu\nu} \theta_\nu) - i \mathfrak{e}_\tau{}^m \left(\tilde{\sigma}_m^{\mu\nu} \dot{\theta}_\nu + \theta^\mu \varepsilon_{mkl} G_\tau{}^{kl} \right) \\
& - 4 \widetilde{\Omega}_{\tau a}{}^a \left[G_\tau{}^{0'm} - 2 \widetilde{\Omega}_{\tau b}{}^b (\theta \sigma^m \bar{\theta}) \right] \tilde{\sigma}_m^{\mu\nu} \theta_\nu + 6 \bar{\theta}^\mu (\Delta_\tau \Delta_\tau + \Omega_{\tau a} \Omega_\tau{}^a) \theta^2 \\
& - \bar{\theta}^\mu G_\tau{}^{0'}{}_m (G_\tau{}^{0'm} + 3 G_\tau{}^{3m}) \theta^2 - 3i \widetilde{\Omega}_{\tau a}{}^a \left(\dot{\bar{\theta}}^\mu \theta^2 - 2\bar{\theta}^\mu (\dot{\theta}\theta) \right) = 0;
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
& - 2i \Phi_L E_{\tau a} \left[\bar{\omega}_\tau{}^{\mu a} + i \bar{\chi}_\tau{}^{\mu a} (\theta\bar{\theta}) \right] - 2i \mathfrak{e}_\tau{}^m c_\tau{}^n (\theta \sigma_n \tilde{\sigma}_m)^\mu + 2i \Delta_\tau \left(\dot{\theta}^\mu - 2i \widetilde{\Omega}_{\tau a}{}^a \theta^\mu \right. \\
& + \left. \tfrac{1}{2} G_\tau{}^{mn} \theta^\nu \sigma_{mn\nu}{}^\mu \right) - 2i \theta^\mu \left(\mathfrak{e}_{\tau m} \mathfrak{e}_\tau{}^m - E_{\tau a} E_\tau{}^a \right) + (\Delta_\tau + 2i \widetilde{\Omega}_{\tau a}{}^a) c_\tau{}^m \tilde{\sigma}_m^{\mu\nu} \bar{\theta}_\nu \theta^2 \\
& + 4 \mathfrak{E}_\tau{}^m (\theta \sigma_m \bar{\theta})(\dot{\theta}^\mu + \tfrac{1}{2} G_\tau{}^{kl} \theta^\nu \sigma_{kl\nu}{}^\mu) + 6 \Delta_\tau G_\tau{}^{0'm} \tilde{\sigma}_m^{\mu\nu} \bar{\theta}_\nu \theta^2 + \tfrac{21i}{2} \Delta_\tau \dot{\theta}^\mu \theta^2 \bar{\theta}^2 = 0
\end{aligned}
\tag{46}
$$

and c.c. When $\theta^\mu = \bar{\theta}^\mu = 0$ these equations coincide with (19).

3.3 Lax representation for $AdS_4 \times \mathbb{CP}^3$ superparticle equations of motion in the partial $\kappa-$symmetry gauge

Similarly to equations of motion (15) for the superparticle on the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset space the system (41)-(46) also admits Lax representation

$$
\frac{dL}{d\tau} + [M, L] = 0
\tag{47}
$$

with M being the same as in (16) and L now taking value in the whole $osp(4|6)$ superalgebra rather than its $g_{(2)}$ eigenspace

$$
L = L_{\text{so}(2,3)} + L_{\text{su}(4)} + L_{24\text{susys}} \in osp(4|6).
\tag{48}
$$

The first term describes anti-de Sitter part of L

$$
L_{\text{so}(2,3)} = 2 \left(\Phi_L E_\tau{}^m - \frac{7}{4} \theta^2 \bar{\theta}^2 G_\tau{}^{0'm} \right) M_{0'm} + \Phi_L \Delta_\tau D + \frac{3}{2} \theta^2 \bar{\theta}^2 G_\tau{}^{0'm} M_{3m} + i (\theta\bar{\theta}) \varepsilon^{kmn} \mathfrak{e}_{\tau k} M_{mn}.
\tag{49}
$$

The second summand in (48) belongs to the $su(4)$ isometry algebra of $\mathbb{CP}^3$ manifold

$$
L_{\text{su}(4)} = -i \Phi_L (E_{\tau a} T^a + E_\tau{}^a T_a) - 4 (\theta \sigma_m \bar{\theta}) E_\tau{}^m \widetilde{V}_a{}^a.
\tag{50}
$$

The contribution proportional to the $osp(4|6)$ fermionic generators

$$
L_{24\text{susys}} = \varepsilon_{(1)}{}^\mu_a Q_{(1)\mu}{}^a + \bar{\varepsilon}_{(1)}{}^{\mu a} \bar{Q}_{(1)\mu a} + \varepsilon_{(3)}{}^\mu_a Q_{(3)\mu}{}^a + \bar{\varepsilon}_{(3)}{}^{\mu a} \bar{Q}_{(3)\mu a},
\tag{51}
$$

where

$$
\varepsilon_{(1)}{}^\mu_a = \varepsilon_{(3)}{}^\mu_a = -\frac{1}{2} E_{\tau a} \bar{\theta}^\mu, \quad \bar{\varepsilon}_{(1)}{}^{\mu a} = \bar{\varepsilon}_{(3)}{}^{\mu a} = \frac{1}{2} E_\tau{}^a \theta^\mu,
\tag{52}
$$

by using (8) can be written simply as

$$
L_{24\text{susys}} = -E_{\tau a} \bar{\theta}^\mu Q_\mu^a + E_\tau{}^a \theta^\mu \bar{Q}_{\mu a}.
\tag{53}
$$

11

Lax connection (48) can be presented as an $osp(4|6)-$valued differential operator acting on the superparticle action (40)

$$
\begin{aligned}
L = &\left(M_{0'm}\frac{\partial}{\partial G_\tau{}^{0'}{}_m} + \frac{1}{2}D\frac{\partial}{\partial\Delta_\tau} - M_{mn}\frac{\partial}{\partial G_{\tau mn}} - M_{3m}\frac{\partial}{\partial G_{\tau 3m}} + T^a\frac{\partial}{\partial\Omega_\tau{}^a} + T_a\frac{\partial}{\partial\Omega_{\tau a}} + \widetilde{V}_a{}^a\frac{\partial}{\partial\widetilde{\Omega}_{\tau b}{}^b} \right. \\
&\left. - \frac{1}{4}Q_{(1)\mu}{}^a\frac{\partial}{\partial\bar\omega_{(3)\tau\mu}{}^a} + \frac{1}{4}\bar{Q}_{(1)\mu a}\frac{\partial}{\partial\omega_{(3)\tau\mu a}} + \frac{1}{4}Q_{(3)\mu}{}^a\frac{\partial}{\partial\bar\omega_{(1)\tau\mu}{}^a} - \frac{1}{4}\bar{Q}_{(3)\mu a}\frac{\partial}{\partial\omega_{(1)\tau\mu a}} \right)\mathscr{S}_{\text{g.f.}}
\end{aligned}
$$

(54)

generalizing corresponding expression (18) for the $OSp(4|6)/(SO(1,3)\times U(3))$ superparticle. We suggest that complete equations of motion for the $AdS_4\times\mathbb{CP}^3$ superparticle that take into account all 8 fermionic coordinates related to the broken space-time supersymmetries are encoded in the same Lax representation (47) with L given by (54), where instead of $\mathscr{S}_{\text{g.f.}}$ the complete action (39) should be substituted.

Let us mention that one can use the freedom in the definition of the Lagrange multiplier $e(\tau)$ in the superparticle action (39) to 'absorb' Φ_L. In the considered partial $\kappa-$symmetry gauge this amounts to the following substitution

$$
-\frac{1}{2}G_\tau{}^{0'}{}_mG_\tau{}^{0'm}\theta^2\bar\theta^2 \rightarrow -\frac{7}{2}G_\tau{}^{0'}{}_mG_\tau{}^{0'm}\theta^2\bar\theta^2
$$

(55)

in the action (40). Corresponding equations of motion and the Lax pair are obtained from (41)-(44) and (48)-(50) by setting $\Phi_L = 1$.

It is interesting to examine how integrable structure of the $AdS_4\times\mathbb{CP}^3$ superparticle is related to that of the superstring. The Lax connection of the $AdS_4\times\mathbb{CP}^3$ superstring $\mathcal{L}_i$, $i = (\tau,\sigma)$, is given by the sum

$$
\mathcal{L}_i = \widehat{\mathcal{L}}_i + *\widetilde{\mathcal{L}}_i,
$$

(56)

where the last term contains $2d$ Hodge dual of $\widetilde{\mathcal{L}}_i$: $*\widetilde{\mathcal{L}}_i = \varepsilon_{ij}\gamma^{jk}\widetilde{\mathcal{L}}_k = \gamma_{ij}\varepsilon^{jk}\widetilde{\mathcal{L}}_k$. Both summands depend on the parameters ℓ_1, ℓ_2, ℓ_3 and ℓ_4 that are the same as those entering the Lax connection of $OSp(4|6)/(SO(1,3)\times U(3))$ sigma-model [7], [8]. They satisfy the constraints

$$
\ell_1^2 - \ell_2^2 = \ell_3\ell_4 = 1, \quad (\ell_1 - \ell_2)\ell_4 = \ell_3, \quad (\ell_1 + \ell_2)\ell_3 = \ell_4
$$

(57)

that can be solved to recover dependence on a single spectral parameter, e.g. as follows

$$
\ell_1 = \frac{1}{2}\left(\frac{1}{z^2} + z^2\right), \quad \ell_2 = \frac{1}{2}\left(\frac{1}{z^2} - z^2\right), \quad \ell_3 = z, \quad \ell_4 = \frac{1}{z}.
$$

(58)

The Lax connection (56) satisfies $2d$ zero curvature condition

$$
\partial_\sigma\mathcal{L}_\tau - \partial_\tau\mathcal{L}_\sigma - [\mathcal{L}_\tau,\mathcal{L}_\sigma] = 0.
$$

(59)

The superparticle limit amounts to dropping the dependence of the superspace coordinate fields on the world-sheet space-like coordinate σ and taking z to unity. Since the Lax connection carries $2d$ vector index only due to the world-sheet derivatives of the superspace coordinate fields it follows that $\widehat{\mathcal{L}}_\sigma = *\widetilde{\mathcal{L}}_\tau = 0$. Using that $\widetilde{\mathcal{L}}_i$ has overall factor of ℓ_2 [29], [30] one can perform the rescaling and define

$$
L = \lim_{z\to 1}\frac{1}{\ell_2}*\widetilde{\mathcal{L}}_\sigma, \quad M = \lim_{z\to 1}\widehat{\mathcal{L}}_\tau
$$

(60)

so that (59) transforms into the Lax representation (47) for the $AdS_4\times\mathbb{CP}^3$ superparticle equations of motion. Reasoning in the opposite direction we conclude that, once the Lax pair for the superparticle is known, this allows to recover dependence of the superstring

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Lax connection on the superspace coordinates (but not the dependence on the spectral parameter) up to terms linear in ℓ_2 for $\widehat{\mathcal{L}}_i$ and quadratic in ℓ_2 for $\widetilde{\mathcal{L}}_i$. For $\widehat{\mathcal{L}}_i$ this just yields the corresponding part of $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model Lax connection since the terms containing Grassmann coordinates related to the broken supersymmetries are proportional to ℓ_2 [29], [30], while for $\widetilde{\mathcal{L}}_i$ the superparticle limit preserves non-trivial information on its dependence on the 'broken' fermions.

4   Conclusion

For superstring models consistent zero-mode limit can be defined that is described by massless superparticles. Superparticle mechanics is in general easier to analyze than that of the corresponding superstring model since there survives only finite number of degrees of freedom. That is why in this paper we addressed Lagrangian mechanics of the massless superparticle on the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold and $AdS_4 \times \mathbb{CP}^3$ superbackground. Because all $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset coordinates can be associated with the (super)symmetries of the manifold, superparticle equations of motion, although non-linear, are expressed in terms of the $osp(4|6)$ Cartan forms and admit the Lax representation implying their integrability.

The case of $AdS_4 \times \mathbb{CP}^3$ superparticle is more involved since the $AdS_4 \times \mathbb{CP}^3$ superspace is additionally parametrized by 8 anticommuting coordinates for the broken supersymmetries and is not isomorphic to a supercoset manifold. It is not apriori obvious that integrable structure of the superparticle/superstring can be extended from the $OSp(4|6)/(SO(1,3) \times U(3))$ supermanifold into the $AdS_4 \times \mathbb{CP}^3$ superspace. First evidence that this is indeed so was given in [29], [30], where the Lax connection of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model had been extended by linear and quadratic contributions of all 8 fermionic coordinates associated with the broken supersymmetries. In Ref. [31] we proposed to use the κ−symmetry gauge freedom to keep in the broken supersymmetries sector less degrees of freedom that facilitates study of the integrable structure beyond the $OSp(4|6)/(SO(1,3) \times U(3))$ supermanifold.

In the present pape we considered the case when 4 fermionic coordinates related to broken part of conformal supersymmetry are gauged away so that there remain 4 coordinates corresponding to broken Poincare supersymmetry. Equations of motion for the $AdS_4 \times \mathbb{CP}^3$ superparticle in such a partial κ−gauge have been derived and shown to be integrable. The Lax pair includes contributions up to the 4th order in the 'broken' fermions. It can be expressed in terms of the differential operator that takes value in the $osp(4|6)$ superalgebra and acts of the superparticle action. We suggest that this form of the Lax pair is generic one for the $AdS_4 \times \mathbb{CP}^3$ superparticle, i.e. does not depend on a particular κ−symmetry gauge choice. Proving it would establish the integrability of the $AdS_4 \times \mathbb{CP}^3$ superparticle and provide further evidence in favor of the integrability of complete equations of motion for the $AdS_4 \times \mathbb{CP}^3$ superstring that is important for identifying its spectrum. Finally we explored the relation between the integrable structures of the superparticle and superstring and found that from the Lax pair of the $AdS_4 \times \mathbb{CP}^3$ superparticle one can gain information on the Hodge dualized part of the superstring Lax connection.

5   Acknowledgements

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